



## YOUNG CHILDREN'S REPRESENTATIONS AND IDEAS ABOUT THE MATHEMATICAL SIGNS OF PLUS AND MINUS

Glykeria Fragkiadaki<sup>i</sup>

Department of Educational Science  
and Early Childhood Education,  
University of Patras,  
Patras, Greece

### Abstract:

Before even entering formal education, young children form mathematical concepts and construct multiple and complex representations and ideas about mathematical symbols as well as about basic mathematical signs. These conceptions, representations, and ideas are critical for their early engagement, learning, and development in Mathematics. Entering school life, children bring into the educational settings these conceptualizations and use them to make sense, joint, participate, and contribute to mathematical educational routines, tasks, and activities. The present study seeks to explore preschool children's representations and ideas about two commonly used in everyday life mathematical symbols: the sign of plus and the sign of minus. The findings of the study give an insight into preschoolers': a) awareness of the signs, b) used terminology, and c) understandings about the role of the signs in daily life. The outcomes of the study inform practice providing a set of suggestions about how signs can be introduced in educational reality in early childhood settings.

**Keywords:** plus sign; minus sign; mathematical symbols; semiotics; mathematical concepts; mathematics; representations; children's ideas; tracing procedure; early childhood

### 1. Introduction

As young children learn and develop in dialectic interrelation with their social and physical environment, they spontaneously and gradually form several concepts including mathematical concepts (Zacharos, Antonopoulos, & Ravanis, 2011). The familiarization with mathematical concepts as well as with the written mathematical language begins from infancy as part of a child's everyday day reality. For example, as infants or toddlers, children join and participate in conversations that mathematical concepts such as "less and more", "shorter and taller", "close and far" are discussed, they

<sup>i</sup> Correspondence: email [gfragkiadaki@upatras.gr](mailto:gfragkiadaki@upatras.gr)

wear clothes and shoes with numbers and signs on, they enter spaces such as pediatrician's office or elevators where graphics and numbering are present, and they engage with objects and materials with diverse mathematical symbols on them such as baby food and nappy packaging. Thus, before even entering formal education children form a complex system of unique representations and ideas about mathematical symbols as well as about basic mathematical signs such as the equal sign, the plus sign, the minus sign. These conceptions, representations, and ideas are critical for their learning and development in Mathematics. When children enter school life, they bring into educational settings these conceptualizations and they use them to make sense, joint, participate, and contribute to educational routines, tasks, and activities. The present study seeks to explore preschool children's representations and ideas about two commonly used signs in everyday life: the sign of plus and the sign of minus. The study begins with an overview of the use of mathematical symbols and signs in early childhood education. This is followed by the presentation of the focus and the methodological choices of the study. The findings of the study give an insight into preschoolers': a) awareness of the signs, b) used terminology, and c) understandings about the role of the signs in daily life. The outcomes of the study inform practice providing a set of suggestions about the approach of the signs in everyday educational reality in early childhood settings.

## **2. The Use of Mathematical Symbols and Signs in Early Childhood Education**

The use of mathematical symbols has a critical role in the development of young children's mathematical thinking and is related to social as well as cultural aspects of mathematics (Steinbring, 2006). Focusing on mathematical knowledge, Alvarado and Ferreiro (2002) argued that the understanding of written mathematical symbols is a complex process that goes through different phases and evolves differently for each child.

The literature review on the conceptualization of basic mathematical symbols by young children has mainly focused on the mathematical symbol of "equal" (Behr et al., 1980; Kieran, 1981; Baroody & Ginsburg, 1983; Rittle-Johnson & Alibali, 1999). What we have learned from the studies in the field is that young children face multiple challenges in understanding the use of the equal sign and teaching and learning interventions are needed to support children on understanding the sign as a cultural and social symbol with special use.

Brizuela and Cayton (2008) also explored the processes of understanding mathematical symbols and in particular punctuation marks such as decimal point (,) and dot (.). What was found is that children, from a very young age and while not taught in formal education, tend to give meaning to them. As Brizuela and Cayton argued, young children use their everyday knowledge and experience to make assumptions and form their understanding of the two signs. In many cases, these assumptions and understandings come in line with the actual use of the signs. The study suggests a

systematic engagement with punctuations, as part of both literacy and numeracy engagement, from early years.

There is a limited number of studies concerning the mathematical symbols of “plus” and “minus”. These are particularly interesting given that these signs are part of young children’s everyday reality at home settings, early childhood, and care settings as well as are present in most of the community activities, for example, shopping at the grocery shop. Linchevski and Williams (1999) had designed a set of teaching and learning interventions aiming at interrelating young children’s everyday concepts and scientific concepts about the positive numbers (e.g., +1) and the negative numbers (e.g., -1). Properly designed tools and models were provided to the students. It was found that to transform children's everyday thinking into “mathematical thinking” it is crucial, to begin with, children's daily understandings and the spontaneous thinking that follows and/or leads these understandings.

However, to achieve the transition of young children’s thinking between everyday understandings to scientific understandings about the use of plus and minus, we need to be aware of how young children conceptualize and think about these signs. More empirical research has to be done in that field.

### **3. The Present Study**

This study aims at tracing and detecting young children’s ideas and representations for the mathematical signs of plus and minus. The study focuses on preschool children's age groups. In line with the aim of the study, the research questions are shaped as follows:

- 1) Do preschoolers recognize the mathematical the plus and minus signs?
- 2) Do preschoolers know the term used for the two signs?
- 3) Do preschoolers know in which situations these signs are used?

The study argues that although the approach of these two mathematical signs is not done systematically in kindergarten, children, having noticed the signs in their wider social and cultural environment such as daily life routines, magazines, packaging, remote controls will have form several and diverse initial representations about the signs.

### **4. Methodological Framework**

#### **4.1 The Study Design**

The tracing procedure was conducted by the technique of the structured interview. The interview took place in two phases. During the first phase, children’s representations and ideas about the symbol of plus were detected. During the second phase, children’s representations, and ideas about the symbol of minus were detected. In each phase, children were engaged with a set of images and figures taken from diverse printed materials such as magazines, newspapers, flyers. Emphasis was given in the use of everyday materials that children were familiar with during daily life. The educators

asked children to observe the sets of images and figures. Then, a set of questions was posed to each child. The questions were formed as follows for each phase of the research procedure: a) "Do you recognize this symbol?", b) "Do you know the name of this symbol?", and c) "Do you know when this symbol is used?". Children were interviewed individually, and each interview lasted between ten (10) to fifteen (15) minutes.

#### 4.2 Participants

The research was conducted at two (2) kindergarten classrooms in an urban area in Greece. Twenty children (20), 11 girls and 9 boys, aged between 4 to 6 years old participated in the study. Parents' consent was given. Children had not participated in systematic teaching and learning interventions during their everyday educational reality at the kindergarten related to the signs of plus and minus. The set of questions were posed by the educators of the classrooms. Children were fully familiarized with the educational routines and the educators.

#### 4.3 Data Collection and Analysis

The collected data were recorded by the educators at forms design specifically for the research procedure. Descriptive statistics were used for the analysis of the collected quantitative data. For the data organization and the description of the results, children's responses were coded and categorized per research question as well as per responses regarding the two signs. The emerged categories are indicative of the number and the type of the variables that were found at the sample. All variables are qualitative and in particular nominal variables. The description of each category is provided in the following subsections. The frequencies and the percentages of the variables are presented.

### 5. Results

The presentation of the results as they emerged from the recording of the children's responses after observing different representations of the signs (Figure 1) follows.

**Figure 1:** The sets of figures and images that were given to the children during the research procedure



Results regarding the plus signs are presented first. This is followed by the presentation of the results regarding the sign of minus. Results are presented separately for each research question.

### A. Regarding the sign of plus

#### a. Do preschoolers recognize the sign of plus?

The following table (Table 1) presents the frequencies and the percentage of the children's responses regarding the research question "Do preschoolers recognize the sign of plus?". Children's responses were classified into two categories: a) negative response in the cases that the children were not able to recognize the sign e.g. "I don't know...", and b) positive response in the cases children were able to recognize the sign e.g., "I have seen that. I know it!".

**Table 1:** Frequencies and percentages for the recognition of the plus sign

Category	Frequency	Percentage (%)
Negative response	7	35
Positive response	13	65
Total	20	100

Regarding the plus sign, it was noticed that the majority of the children (13/20) seem to recognize it as a familiar sign e.g., "I have seen that before!". However, there is a correspondingly large percentage of the sample (7/20) not to recognize it.

#### b. Do preschoolers know the term used for the sign of plus?

The following table (Table 2) presents the frequencies and the percentages of the children's responses regarding the research question "Do preschoolers know the term used for the sign of plus?". Children's responses were classified into six categories: a) No term used in the cases that children did not name the sign, b) Plus in the cases that children named the sign as "plus", c) Cross in cases children mention a term related with religion e.g., "When a church is built, we put this. It is a cross.", d) Hospital/ Doctor in cases children mention a term related to medical experience e.g., "It says "Doctor".", e) Letter in cases children mentioned an alphabet letter e.g., "Letter! We use it to make people write their name", and f) sign in case children mention that it is called sign using the same term the educators used.

According to the data in the table below (Table 2), most children (6/20) either recognized the sign as a "cross" or as an alphabet letter (6/20). It is also worth noting that one (1) child used the term "plus".

**Table 2:** Frequencies and percentages of terms used for the plus sign

Category	Frequency	Percentage (%)
No term used	4	20
Plus	1	5
Cross	6	30
Hospital/ Doctor	2	10
Letter	6	30
Sign	1	5
Total	20	100

c. Do preschoolers know in which situations the plus sign is used?

The following table (Table 3) presents the frequencies and the percentages of the children's responses regarding the research question "Do preschoolers know in which situations the plus sign is used?". Children's responses were classified into six categories: a) Negative response in cases children did not express any ideas about the use of the sign, b) Relation to mathematics in cases children mentioned a use related to mathematical concepts e.g., "To count the numbers correctly", c) Relation to institutional ensign in cases children mentioned e.g., "For the dentist. To take out your teeth!", d) Relation to the writing process in cases children related the sign with the writing process, and e) Relation to marking in cases children mentioned that it was used as a marking sign e.g. "To mark your things".

**Table 3:** Frequencies and percentages for the use of the plus sign

Category	Frequency	Percentage (%)
Negative response	7	35
Relation to mathematics	2	10
Relation to institutional ensign	6	30
Relation to the writing process	3	15
Relation to marking	2	10
Total	20	100

The responses children gave to this question were closely related to their responses to the first question "Do you know the name of this symbol?". It is observed that most children seemed to either not know in which cases the specific sign is used (7/20) or to have alternative representations (6/20). Of particular interest, however, is the answer of two (2) children (category "related to mathematics") who stated: "to increase the numbers, they multiply, they become more, I know this from Bob The Sponge", "For counting the numbers correctly!").

## B. Regarding the sign of minus

### a. Do preschoolers recognize the sign of minus?

The following table (Table 4) presents the frequencies and the percentages of the children's responses regarding the research question "Do preschoolers recognize the sign of minus?". Children's responses were classified into two categories: a) negative response in the cases that the children were not able to recognize the sign e.g. "I don't know...", and b) positive response in the cases children were able to recognize the sign e.g., "I know it!".

**Table 4:** Frequencies and percentages for the recognition of the minus sign

Category	Frequency	Percentage (%)
Negative response	8	40
Positive response	12	60
Total	20	100

Regarding the minus sign, it was noticed that the majority of children (12/20) seemed to recognize it as a familiar sign. However, there is a correspondingly large percentage of the sample (8/20) not to recognize it.

### b. Do preschoolers know the term used for the sign of minus?

The following table (Table 5) presents the frequencies and the percentages of the children's responses regarding the research question "Do preschoolers know the term used for the sign of minus?". Children's responses were classified into seven categories: a) No term used in the cases that children did not name the sign, b) Minus in the cases that children named the sign as "minus", c) Other mathematical terms in cases children mention a mathematical term but not the minus e.g. "Equal.", d) Highway Code in cases children mention a term related with Highway Code e.g., "It says "Stop".", e) Letter in cases children mentioned an alphabet letter e.g., "The letter "X" (in Greek is written "Ξ")", f) punctuation mark in case children mention that it is used for writing "Like the dot", and g) Geometrical shape and figure e.g. "A line."

**Table 5:** Frequencies and percentages for the term used of the plus sign

Category	Frequency	Percentage (%)
No term used	9	45
Minus	1	5
Other mathematical terms used	1	5
Highway Code	1	5
Letter	2	10
Punctuation mark	1	5
Geometrical shape & figure	5	25
Total	20	100

Regarding the term used for the minus sign, the data at the above table (Table 5) show that a high percentage of the children (9/20) does not use any term to name the sign. Five (5) out of eleven (11) children named the sign using a term related to geometrical shapes of figures.

c. Do preschoolers know in which situations the minus sign is used?

The following table (Table 6) presents the frequencies and the percentages of the children's responses regarding the research question "Do preschoolers know in which situations the minus sign is used?". Children's responses were classified as follows: a) Negative response in cases children did not express any ideas about the use of the sign, b) Relation to mathematics in cases children mentioned a use related to mathematical concepts e.g., "To count the numbers correctly", c) Relation to the Highway Code in cases children mentioned e.g., "When you drive your car", d) Relation to the writing process in cases children related the sign with the writing process e.g., "We use it to write the letter "X" (in Greek is written "Ξ").

**Table 6:** Frequencies and percentages for the use of the minus sign

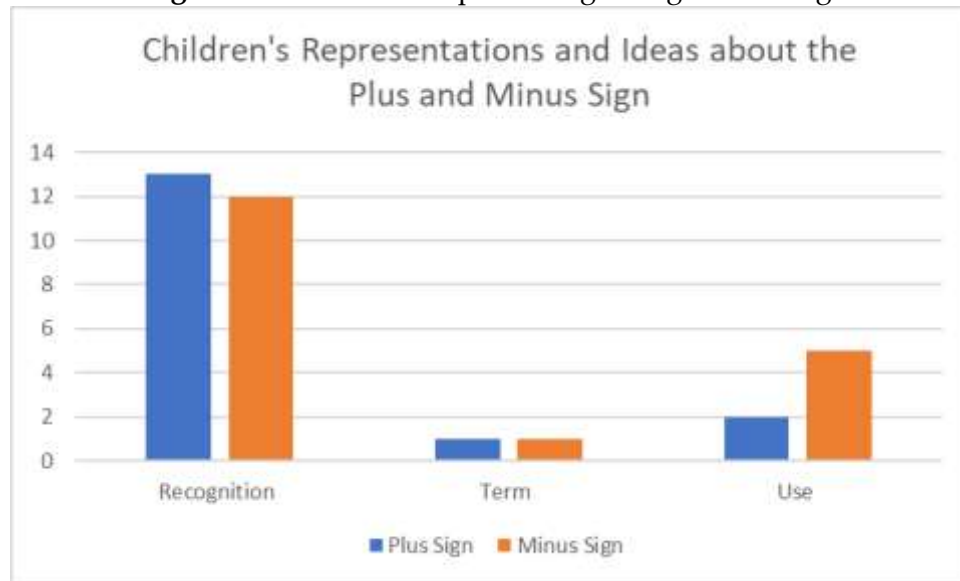
Category	Frequency	Percentage (%)
Negative response	9	45
Relation to mathematics	5	25
Relation with the Highway Code	1	5
Relation to the writing	5	25
Total	20	100

Regarding the use of minus, we observe that a high percentage of the children (9/20) in the sample do not know when the sign is used. Of particular interest is the fact that the percentage of children (5/20) who have math-related responses is quite high.

The below diagram illustrates the children's responses by each research question and by each symbol. Studying the diagram, it is observed that the children in the sample show almost equal percentages in the recognition of the symbols. Children faced equally considerable difficulties in naming the sign. Determining the use of the sign has been more difficult in the case of the plus sign. This fact can be explained by the qualitative analysis of our data. As it turns out, children were familiar with the minus sign from their experience with product packaging and the inscriptions in the grocery stores where discounts on prices are indicated using the minus sign.



**Diagram 1:** Children's responses regarding the two signs



## 6. Discussion and Conclusions

The present study explored preschool children's representations and ideas about two commonly used signs in everyday life: the sign of plus and the sign of minus. The findings of the study gave an important insight into preschoolers': a) awareness of the signs, b) used terminology, and c) understandings about the role of the signs in daily life. What was found is shaped as follows:

- The majority of the children that participated in the study were aware of the two mathematical signs and were able to recognize them both,
- Children were almost at the same extent aware of the plus sign as the minus sign,
- The majority of the children were not able to use the appropriate term to refer to the two signs,
- Children faced difficulties in defining the use of both signs, and
- Children were more aware of the use of the minus sign than the use of the plus sign.

The outcomes of the study are indicative of the fact that preschool children can observe, identify, and make assumptions about the two mathematical signs long before the systematical approach of the signs into the framework of formal education. Although the term or usage of the signs is not familiar to them, their symbolic dimension and their meaningful role are spontaneously recognized. This finding highlights the dynamics of signs as well as the critical role of diverse forms of symbols and semiotics in early years Science, Technology, Engineering, and Mathematics education (Delserieys-Pedregosa, Impedovo, Fragkiadaki, & Kampeza, 2017; Fragkiadaki, Fleer, & Ravanis, 2017; Herakleioti & Pantidos, 2016; Kampeza, Vellopoulou, Fragkiadaki, & Ravanis, 2016; Papandreou, 2009).

What is also important to be noted here is that the capacity or not of children to be aware of the signs, use valid terminology, and understand their use should not be related

to the capacity of children to understand and solve addition and subtraction. As Canobi and Bethune (2008) pointed out, children can do additions and subtractions before being able to translate the physical effects of these operations into number words at a symbolic level. That means that the ability of young children to engage with addition and abstraction at a physical level is not dialectical interrelated or depends on the engagement at a symbolic level.

The outcomes of the study inform practice providing suggestions about the approach of the plus and minus signs in everyday educational reality in early childhood settings. The understanding of the semiotics that lies behind the mathematical signs has to be conceptualized as a unique and distinct learning experience in early learning in mathematics. That requires educational and pedagogical planning and includes concrete teaching goals and anticipated learning outcomes. Teachers can organize teaching and learning tasks based on the recognition of the signs in the wider social and cultural context. This can be followed by the understanding of the use of signs as an abstract written language and the functionality that comes with it, and finally the introduction of the appropriate terms. As Carruthers and Worthington (2008) pointed out the pedagogy of early written mathematics needs to be better understood and developed. Further research work needs to be done to enhance early mathematics pedagogy and inform policy.

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